Streamer Simulations With Highly Accurate Transport Data

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Positive streamers in N_2:O_2-mixtures, like air, have been found to be remarkably robust against changes in the source of free electrons in front of the streamer head and the N_2-O_2 ratio, both experimentally and in simulations. However, the influence of more accurate transport data (diffusion and mobility coefficients) has not yet been investigated. In this work, we recall recent results in simulations of positive streamers in N_2:O_2 mixtures and their comparison with experiments, we describe a strategy to obtain accurate field-dependent transport data and finally provide some initial results obtained with these data. Simulations were done for streamers in air at standard temperature and pressure, but transport data can be generated for other pressures and gas compositions.

1. Introduction

Streamers are thin channels of ionized gas that occur in the initial stages of lightning, in sprite-discharges above thunderclouds and in industrial applications such as lighting and gas cleaning. We distinguish between positive and negative streamers, where negative streamers propagate with the electron drift velocity while positive streamers propagate against. Nevertheless, positive streamers emerge more easily and propagate faster than negative ones.

Streamers can be simulated with particle models, where the motion of individual partciles are followed with Monte Carlo techniques or with fluid models, that approximate the physical particles as a continuous density and solve the drift-diffusion-reaction equations for density fluxes. In both models, the electric field is coupled via Poisson's equation. In fluid modelling, diffusion and mobility coefficients as well as reaction rates are gas-dependent variables that can influence streamer propagation.

2. Previous results

In previous work we have focused on the effect of photo-ionization, background ionization and gas composition with constant mobility and diffusion coefficients [1]. Using our fluid code with adaptive grid refinement described in [2], we have compared photo-ionization and background ionization as sources of free electrons in front of the streamer head [3] as well as varying N_2:O_2 ratios. We concluded that the velocities and diamters of streamers are remarkably insensitive to the source and quantity of free electrons in front of the streamer head. Changing the background ionization density by 2 orders of magnitude resulted in a difference of only 20% in the time it takes for the streamer to cross the electrode gap, as can be seen in figure 1.

In addition, we found that even in nitrogen with 1 ppm oxygen, the photo-ionization mechanism still produced sufficiently many free electrons for positive streamers to propagate. These results were compared with experimental observations [4]. In experiments, the morphology of streamers in air was noticably different from streamers in nitrogen as streamers in air appeared to be smooth channels, while streamers in near-pure nitrogen have a feather-like structure with small hairs connected to the main streamer channel. In addition, streamers in nitrogen branch more often than those in air. Using the results from streamer simulations we provided an explanation for the feather-like structures in streamers in nitrogen and their absence in streamers in air [5].

3. Calculation of transport data

Electron transport coefficients for fluid modeling of streamers are derived from the non-conservative Boltzmann equation

$$\frac{\partial f}{\partial t} + \boldsymbol{c} \cdot \frac{\partial f}{\partial \boldsymbol{r}} + \frac{q}{m} \boldsymbol{E} \cdot \frac{\partial f}{\partial \boldsymbol{c}} = -J(f, f_o) \,. \qquad (1)$$

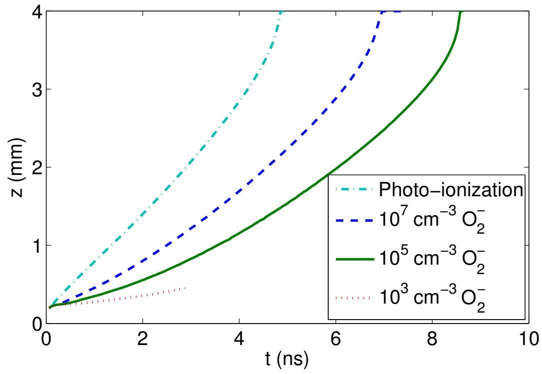

Fig. 1: Simulation of positive streamers in air at standard temperature and pressure. Position of streamer head is plotted as a function of time for streamers with photo-ionization (top curve) or background ionization (bottom 3 curves, level of O_2^- indicated in legend). Streamers in an needle-plane electrode configuration with 12 kV potential over a 4 mm gap.

Here $f(\boldsymbol{r}, \boldsymbol{c}, t)$ is the phase-space distribution function, $\boldsymbol{r}$ and $\boldsymbol{c}$ denote the position and velocity coordinates, q and m are the charge and mass of the electrons, t is time and E is the electric field. The right hand side of (1) denotes the linear electron-neutral molecule collision operator, accounting for elastic, inelastic, and non-conservative collisions. Solution of the Boltzmann equation for electrons and light ions under the influence of a spatially-homogeneous electric field has been recently detailed by Dujko et al. [6] and we emphasize here only the following critical points:

(1) Spherical harmonic and Sonine polynomial expansions. The directional dependence of $f(\boldsymbol{r}, \boldsymbol{c}, t)$ in velocity space is represented in terms of a spherical harmonic expansion:

$$f(\boldsymbol{r}, \boldsymbol{c}, t) = \sum_{l=0}^{\infty} \sum_{m=0}^{\infty} f_m^{(l)}(\boldsymbol{r}, c, t) Y_m^{[l]}(\hat{\boldsymbol{c}}) , \qquad (2)$$

where $Y_m^{[l]}(\hat{\boldsymbol{c}})$ are spherical harmonics and $\hat{\boldsymbol{c}}$ denotes the angles of $\boldsymbol{c}$. No restrictions are placed on the number of spherical harmonics nor is any particular form of the time-dependence of the expansion coefficients assumed. The speed dependence of the coefficients $f(\boldsymbol{r}, \boldsymbol{c}, t)$ is treated by an expansion about a Maxwellian at an arbitrary time-dependent basis temperature in terms of Sonine polynomials.

(2) Density gradient expansion. It is assumed that the hydrodynamic stage has been reached and that spatial dependence is treated by the density gradient expansion:

$$f(\boldsymbol{r}, \boldsymbol{c}, t) = \sum_{k=0}^{\infty} f^{(k)}(\boldsymbol{c}, t) \odot (-\nabla)^k n(\boldsymbol{r}, t) \ . \qquad (3)$$

Using the above decompositions of f and an implicit finite difference evaluation of time derivatives, the Boltzmann equation is transformed into a hierarchy of doubly infinite coupled inhomogeneous matrix equations for the time-dependent moments. Finite truncation of both the Sonine polynomial and spherical harmonic expansions permit solution of this hierarchy by direct numerical inversion. Having obtained the moments, the transport coefficients and other transport properties can be calculated and their explicit expressions are given in [6].

We consider the reduced electric field range: 1-1000 Td (1 Td = 10^{-21} Vm^2). A neutral temperature of 300 K is assumed and superelastic collisions are not allowed for. The cross sections for the electron scattering in N_2 detailed by Stojanović and Petrović [7] and cross sections for electron scattering in O_2 developed by Itikawa et al. [8, 9] are implemented in this work. Figs. 2 and 3 show mobility and diffusion coefficient for electrons in a $N_2 : O_2 = 80 : 20$ air-like mixture needed as input for the solution of the classical drift-diffusion eqs. [3].

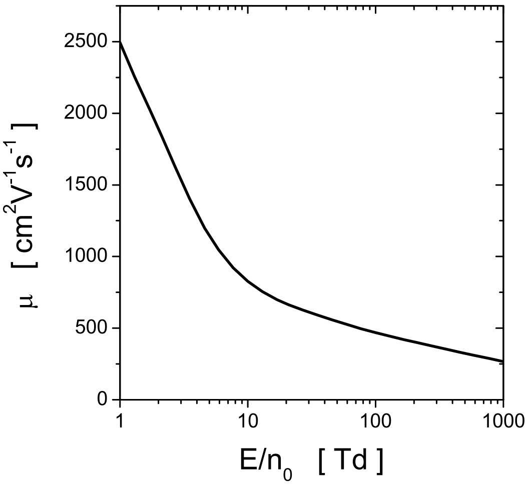

Fig. 2: Electron mobility as a function of E/n_0 in a $N_2 : O_2 = 80 : 20$ air-like mixture.

4. Preliminary results and conclusion

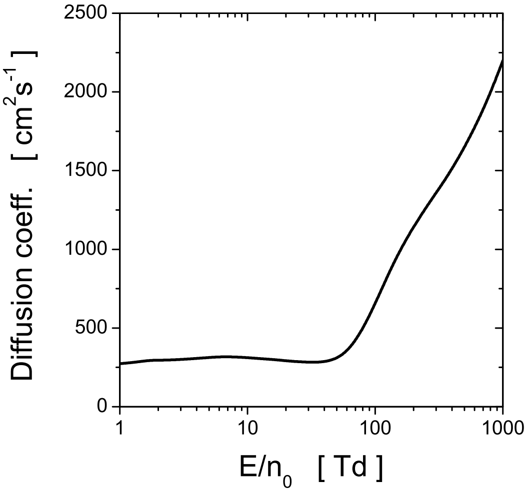

Fig. 3: Diffusion coefficient for electrons as a function of E/n_0 in a $N_2 : O_2 = 80 : 20$ air-like mixture.

We have simulated positive streamers using these transport data, leaving other parameters and reaction rates unchanged. Figure 4 shows that the field-dependent mobility and diffusion coefficients have a significant effect on streamer velocities. The simulated streamers using the field-dependent transport data are slower than those with the old coefficients.

The streamers with the new transport data appear to be slightly thinner than those with the old coefficients. In addition, the field-dependent transport data yield streamers with a lower maximum electric field, by about 10%. The field-dependent mobility coefficient is higher than the constant value previously used at low field values (below 50 kV/cm). Because of this, and because the photo-ionization rate is lower due to the lower maximum electric field in the streamer head, we see that the electron density in front of the streamerhead is lower in the simulations with the field-dependent transport data as less electrons are created by photo-ionization and the electrons drift into the streamer channel more rapidly due to the higher mobility in the background field.

At the time of writing, these results are preliminary and additional simulations are being performed. These results will be presented and discussed at the conference. But for now, we can already conclude that different transport data affect streamer simulations more than different data for photo-ionization and background ionization. With the means to obtain accurate transport data, we can now investigate streamer propagation in a variety of gas compositions and pressures, such as planetary atmospheres of Venus and Jupiter.

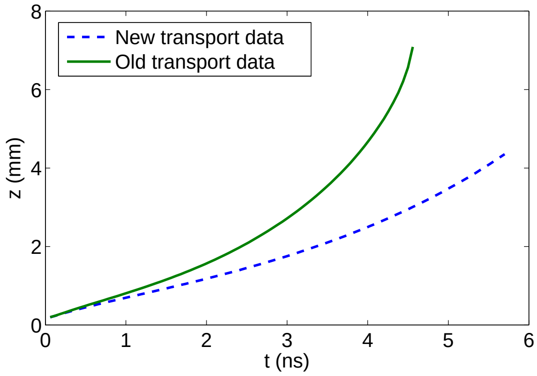

Fig. 4: Simulation of positive streamers in air at standard temperature and pressure. The position of streamer head is plotted as a function of time for streamers with old transport coefficients (solid curve) and newtransport data (dashed curve). Streamers in a needle-plane electrode configuration with 24 kV potential over an 8 mm gap.

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